Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Sequence & Series

***Dream is not that which you see while sleeping; it is something that does not let you sleep.-A.P.J. Abdul Kalam*

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. Number of identical terms in the sequence 2, 5, 8, 11, ... upto 100 terms and 3, 5, 7, 9, 11, ... upto 100 terms are

(a) 17 (b) 33
(c) 50 (d) 147

2. If $ab^2c^3, a^2b^3c^4, a^3b^4c^5$ are in AP $(a, b, c > 0)$, then the minimum value of $a + b + c$ is

(a) 1 (b) 3
(c) 5 (d) 9

3. If $21(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$, then x, y, z are in

(a) AP (b) GP
(c) HP (d) not in AP/GP/HP

4. The sum of the integers lying between 1 and 100 (both inclusive) and divisible by 3 or 5 or 7 is

(a) 818 (b) 1828
(c) 2838 (d) 3848

5. The maximum value of the sum of the AP 50, 48, 46, 44, ... is

(a) 648 (b) 450
(c) 558 (d) 650

6. If $a_i > 0, i = 1, 2, 3, \dots, 50$ and $a_1 + a_2 + a_3 + \dots + a_{50} = 50$, then the minimum value of $\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_{50}}$ is equal to

(a) 150 (b) 100
(c) 50 (d) $(50)^2$

7. If positive numbers a^{-1}, b^{-1}, c^{-1} are in AP, then the product of roots of the equation $x^2 - kx + 2b^{101} - a^{101} - c^{101} = 0$, $(k \in R)$ has

(a) > 0 (b) < 0
(c) $= 0$ (d) undefined

8. If $x^a = y^b = z^c$, where a, b, c are unequal positive numbers and x, y, z are in GP, then $a^3 + c^3$ is

(a) $\geq 2b^3$ (b) $> 2b^3$
(c) $\leq 2b^3$ (d) $< 2b^3$

9. The 1025th term in the sequence 1, 22, 4444, 88888888, ... is

(a) 2^9 (b) 2^{10}
(c) 2^{11} (d) 2^{12}

10. If a, b, c are non-zero real numbers such that $3(\Sigma a^2 + 1) = 2(\Sigma a + \Sigma ab)$, then a, b, c are in

(a) AP (b) GP
(c) HP (d) AP and GP

11. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $m = (a + b)(c + d)$ satisfies the relation

(a) $0 < m \leq 1$ (b) $1 \leq m \leq 2$
(c) $2 \leq m \leq 3$ (d) $3 < m \leq 4$

12. If $1 + \lambda + \lambda^2 + \dots + \lambda^n = (1 + \lambda)(1 + \lambda^2)(1 + \lambda^4)(1 + \lambda^8)(1 + \lambda^{16})$, then the value n is (where $n \in N$)

(a) 32 (b) 16
(c) 31 (d) 15

13. If $a_1, a_2, a_3, \dots, a_{20}$ are AMs between 13 and 67, then the maximum value of $a_1 a_2 a_3 \dots a_{20}$ is

(a) $(20)^{20}$ (b) $(40)^{20}$
(c) $(60)^{20}$ (d) $(80)^{20}$

14. If $a_1, a_2, \dots, a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \dots + a_{n-1} + 2a_n$ is

(a) $n(2c)^{1/n}$ (b) $(n+1)c^{1/n}$
(c) $2nc^{1/n}$ (d) $(n+1)(2c)^{1/n}$

15. Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP. If $a > b > c$ and $a + b + c = 3/2$, then the value of a is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2} + \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} + \frac{1}{\sqrt{2}}$

16. If $\alpha \in (0, \pi/2)$, then $\sqrt{(x^2+x)} + \frac{\tan^2\alpha}{\sqrt{(x^2+x)}}$ is always greater than or equal to
(a) $2\tan\alpha$ (b) 1
(c) 2 (d) $\sec^2\alpha$

17. If a, b, c, d are distinct integers in AP such that $d = a^2 + b^2 + c^2$, then $a + b + c + d$ is
(a) 0 (b) 1
(c) 2 (d) 3

18. An infinite GP has first term x and sum 5, then x belongs to
(a) $x < -10$ (b) $-10 < x < 0$
(c) $0 < x < 10$ (d) $x > 10$

19. If $\sum_{i=1}^{21} a_i = 693$, where $a_1, a_2, \ldots, a_{21}$ are in AP, then the value of $\sum_{r=0}^{10} a_{2r+1}$ is
(a) 361 (b) 363
(c) 365 (d) 398

20. In the quadratic equation $ax^2 + bx + c = 0$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in GP, where α, β are the roots of $ax^2 + bx + c = 0$, then
(a) $\Delta \neq 0$ (b) $b\Delta = 0$
(c) $c\Delta = 0$ (d) $\Delta = 0$

21. If a_1, a_2, a_3, a_4 are in HP, then $\frac{1}{a_1 a_4} \sum_{r=1}^{3} a_r a_{r+1}$ is a root of
(a) $x^2 + 2x + 15 = 0$ (b) $x^2 + 2x - 15 = 0$
(c) $x^2 - 2x + 15 = 0$ (d) $x^2 - 2x - 15 = 0$

22. If a, b, c are in AP, then $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in
(a) AP (b) GP
(c) HP (d) no definite sequence

23. If a, b, c, d, e, f are in AP, then $(e - c)$ is equal to
(a) $2(c - a)$ (b) $2(d - b)$
(c) $2(f - d)$ (d) $2(d - c)$

24. If $\log_3 2, \log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ are in AP, then the value of x is
(a) 2 (b) 3
(c) 4 (d) 5

25. If the ratio of the sums of m and n terms of an AP, is $m^2 : n^2$, then the ratio of its mth and nth terms is
(a) $(m-1):(n-1)$
(b) $(2m+1):(2n+1)$
(c) $(2m-1):(2n-1)$
(d) none of the above

26. If the sum of first n positive integers is $\frac{1}{5}$ times the sum of their squares, then n equals
(a) 5 (b) 6
(c) 7 (d) 8

27. The interior angles of a polygon are in AP the smallest angle is 120° and the common difference is 5°. Then, the number of sides of polygon, is
(a) 5 (b) 7
(c) 9 (d) 15

28. If a, b, c are in AP, then the equation $(a-b)x^2 + (c-a)x + (b-c) = 0$ has two roots which are
(a) rational and equal (b) rational and distinct
(c) irrational conjugates (d) complex conjugates

29. If the sum of first n terms of an AP is $(Pn + Qn^2)$, where P, Q are real numbers, then the common difference of the AP is
(a) $P - Q$ (b) $P + Q$
(c) $2Q$ (d) $2P$

30. Given two numbers a and b. Let A denote their single AM and S denote the sum of n AM's between a and b, then (S/A) depends on
(a) n, a, b (b) n, a
(c) n, b (d) n only

31. If $x, (2x+2), (3x+3), \ldots$ are in GP, then the next term of this sequence is
(a) 27 (b) -27
(c) 13.5 (d) -13.5

32. If each term of a GP is positive and each term is the sum of its two succeeding terms, then the common ratio of the GP is
(a) $\left(\frac{\sqrt{5}-1}{2}\right)$ (b) $\left(\frac{\sqrt{5}+1}{2}\right)$
(c) $-\left(\frac{\sqrt{5}+1}{2}\right)$ (d) $\left(\frac{1-\sqrt{5}}{2}\right)$

33. The largest interval for which the series $1 + (x-1) + (x-1)^2 + \ldots\infty$ may be summed, is
(a) $0 < x < 1$ (b) $0 < x < 2$
(c) $-1 < x < 1$ (d) $-2 < x < 2$

34. Three numbers, the third of which being 12, form decreasing GP. If the last term were 9 instead of 12, the three numbers would have formed an AP. The common ratio of the GP is
(a) 1/3 (b) 2/3
(c) 3/4 (d) 4/5

35. The coefficient of x^{49} in the product $(x-1)(x-3)\ldots(x-99)$ is
(a) -99^2 (b) 1
(c) -2500 (d) none of these

36. If $(1.05)^{50} = 11.658$, then $\sum_{n=1}^{49} (1.05)^n$ equals
(a) 208.34 (b) 212.12
(c) 212.16 (d) 213.16

37. If a, b, c are digits, then the rational number represented by $0.c\dot{a}ba\dot{b}\ldots$ is
(a) $cab/990$ (b) $(99c + ba)/990$
(c) $(99c + 10a + b)/99$ (d) $(99c + 10a + b)/990$

38. If $\log_2(a+b)+\log_2(c+d)\geq 4$. Then the minimum value of the expression $a+b+c+d$ is

(a) 2 (b) 4
(c) 8 (d) none of these

39. The HM of two numbers is 4 and their AM and GM satisfy the relation $2A+G^2=27$, then the numbers are

(a) -3 and 1 (b) 5 and -25
(c) 5 and 4 (d) 3 and 6

40. If $\Sigma n=55$, then Σn^2 is equal to

(a) 385 (b) 506
(c) 1115 (d) 3025

41. The natural numbers are grouped as follows $\{1\}, \{2, 3, 4\}, \{5, 6, 7, 8, 9\}, \ldots$, then the first element of the nth group is

(a) n^2-1 (b) n^2+1
(c) $(n-1)^2-1$ (d) $(n-1)^2+1$

42. A monkey while trying to reach the top of a pole of height 12 m takes every time a jump of 2 m but slips 1 m while holding the pole. The number of jumps required to reach the top of the pole, is

(a) 6 (b) 10
(c) 11 (d) 12

43. The sum of the series $1\cdot n+2\cdot(n-1)+3\cdot(n-2)+\ldots+n\cdot 1$ is

(a) $\dfrac{n(n+1)(n+2)}{6}$ (b) $\dfrac{n(n+1)(n+2)}{3}$
(c) $\dfrac{n(n+1)(2n+1)}{6}$ (d) $\dfrac{n(n+1)(2n+1)}{3}$

44. If p, q, r are three positive real numbers are in AP, then the roots of the quadratic equation $px^2+qx+r=0$ are all real for

(a) $\left|\dfrac{r}{p}-7\right|\geq 4\sqrt{3}$ (b) $\left|\dfrac{p}{r}-7\right|<4\sqrt{3}$
(c) all p and r (d) no p and r

45. If $\dfrac{a+bx}{a-bx}=\dfrac{b+cx}{b-cx}=\dfrac{c+dx}{c-dx}$ $(x\neq 0)$, then a, b, c, d are in

(a) AP (b) GP
(c) HP (d) none of these

46. Two AMs A_1 and A_2; two GMs G_1 and G_2 and two HMs H_1 and H_2 are inserted between any two numbers, then $H_1^{-1}+H_2^{-1}$ equals

(a) $A_1^{-1}+A_2^{-1}$ (b) $G_1^{-1}+G_2^{-1}$
(c) $G_1G_2/(A_1+A_2)$ (d) $(A_1+A_2)/G_1G_2$

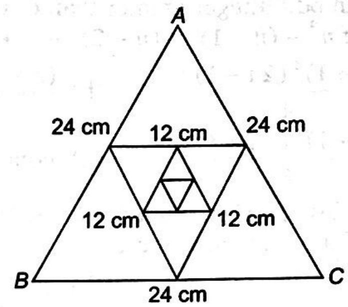
47. The sum of the products of ten numbers $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ taking two at a time is

(a) -55 (b) 55
(c) 165 (d) -165

48. Given that n arithmetic means are inserted between two sets of numbers $a, 2b$ and $2a, b$, where $a, b\in R$. Suppose further that mth mean between these two sets of numbers is same, then the ratio, $a:b$ equals

(a) $n-m+1:m$ (b) $n-m+1:n$
(c) $m:n-m+1$ (d) $n:n-m+1$

49. One side of an equilateral triangle is 24 cm. The mid points of its sides are joined to form another triangle whose mid points are in turn jointed to form still another triangle. This process continues infinitely. The sum of the perimeters of all the triangles is



(a) 144 cm (b) 169 cm
(c) 400 cm (d) 625 cm

50. If $\dfrac{a+b}{1-ab}, b, \dfrac{b+c}{1-bc}$ are in AP, then $a, \dfrac{1}{b}, c$ are in

(a) AP (b) GP
(c) HP (d) none of these

51. If $a_1, a_2, a_3, \ldots, a_n$ are in HP and $f(k)=\sum_{r=1}^{n} a_r - a_k$, then $\dfrac{a_1}{f(1)}, \dfrac{a_2}{f(2)}, \dfrac{a_3}{f(3)}, \ldots, \dfrac{a_n}{f(n)}$ are in

(a) AP (b) GP
(c) HP (d) none of these

52. The sixth term of an AP is equal to 2. The value of the common difference of the AP which makes the product $a_1a_4a_5$ least is given by

(a) $\dfrac{8}{5}$ (b) $\dfrac{5}{4}$
(c) $\dfrac{2}{3}$ (d) none of these

53. The solution of $\log_{\sqrt{3}} x+\log_{\sqrt[4]{3}} x+\log_{\sqrt[6]{3}} x+\ldots+\log_{\sqrt[16]{3}} x=36$ is

(a) $x=3$ (b) $x=4\sqrt{3}$
(c) $x=9$ (d) $x=\sqrt{3}$

54. If $a_1, a_2, a_3, \ldots, a_n$ are in HP, then $\dfrac{a_1}{a_2+a_3+\ldots+a_n}, \dfrac{a_2}{a_1+a_3+\ldots+a_n}, \ldots, \dfrac{a_n}{a_1+a_2+\ldots+a_{n-1}}$ are in

(a) AP (b) GP
(c) HP (d) AGP

55. If the arithmetic progression whose common difference is none zero, the sum of first $3n$ terms is equal to the sum of the next n terms. Then the ratio of the sum of the first $2n$ terms to the next $2n$ terms is

(a) 1/5 (b) 2/3
(c) 3/4 (d) none of these

56. Let a, b, c be three positive prime numbers. The progression in which $\sqrt{a}, \sqrt{b}, \sqrt{c}$ can be three terms (not necessarily consecutive) is

(a) AP (b) GP

(c) HP (d) none of these

57. If n is an odd integer greater than or equal to 1, then the value of $n^3 - (n-1)^3 + (n-2)^3 - \ldots + (-1)^{n-1} 1^3$ is

(a) $\frac{(n+1)^2(2n-1)}{4}$ (b) $\frac{(n-1)^2(2n-1)}{4}$

(c) $\frac{(n+1)^2(2n+1)}{4}$ (d) none of these

58. If the sides of a right angled triangle form an AP, then the sines of the acute angles are

(a) $\frac{3}{5}, \frac{4}{5}$ (b) $\sqrt{3}, \frac{1}{3}$

(c) $\sqrt{\left(\frac{\sqrt{5}-1}{2}\right)}, \sqrt{\left(\frac{\sqrt{5}+1}{2}\right)}$ (d) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

59. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3)\ldots(x-n)$ is

(a) $\frac{n(n^2+2)(3n+1)}{24}$ (b) $\frac{n(n^2-1)(3n+2)}{24}$

(c) $\frac{n(n^2+1)(3n+4)}{24}$ (d) none of these

60. Let $\{a_n\}$ be a GP such that $\frac{a_4}{a_6} = \frac{1}{4}$ and $a_2 + a_5 = 216$. Then a_1 is equal to

(a) 12 or $\frac{108}{7}$ (b) 10

(c) 7 or $\frac{54}{7}$ (d) none of these

61. If $< a_n >$ is an AP and $a_1 + a_4 + a_7 + \ldots + a_{16} = 147$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to

(a) 96 (b) 98

(c) 100 (d) none of these

62. The sum to infinity of the series,

$1 + 2\left(1 - \frac{1}{n}\right) + 3\left(1 - \frac{1}{n}\right)^2 + \ldots$ is

(a) n^2 (b) $n(n+1)$

(c) $n\left(1 + \frac{1}{n}\right)^2$ (d) none of these

63. If $a_1, a_2, \ldots, a_n$ are n non zero real numbers such that $(a_1^2 + a_2^2 + \ldots + a_{n-1}^2)(a_2^2 + a_3^2 + \ldots + a_n^2)$ $\le (a_1 a_2 + a_2 a_3 + \ldots + \ldots + a_{n-1} a_n)^2$, then $a_1, a_2 \ldots, a_n$ are in

(a) AP (b) GP

(c) HP (d) none of these

64. The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3), \ldots$, then sum of the numbers in the nth group is

(a) $\frac{1}{8} n^3 (n^2+1)(n^2+3)$

(b) $\frac{1}{16} n^3 (n^2+16)(n^2+12)$

(c) $\frac{n^3}{12}(n^2+2)(n^2+4)$

(d) none of the above

65. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in AP such that $|a| < 1, |b| < 1$ and $|c| < 1$, then x, y, z are in

(a) AP (b) GP

(c) HP (d) none of these

66. Let a_n be the nth term of the GP of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \ne \beta$, then the common ratio is

(a) α/β (b) β/α

(c) $\sqrt{(\alpha/\beta)}$ (d) $\sqrt{(\beta/\alpha)}$

67. If $a_1 = 0$ and $a_1, a_2, a_3, \ldots, a_n$ are real numbers such that $|a_i| = |a_{i-1} + 1|$ for all i, then the arithmetic mean of the numbers $a_1, a_2, \ldots, a_n$ has value x where

(a) $x < -1$ (b) $x < -\frac{1}{2}$

(c) $x \ge -\frac{1}{2}$ (d) $x = -\frac{1}{2}$

68. If a_1, a_2, a_3 $(a_1 > 0)$ are in GP with common ratio r, then the value of r for which the inequality $9a_1 + 5a_3 > 14a_2$ holds can not lie in the interval

(a) $[1, \infty)$ (b) $\left[1, \frac{9}{5}\right]$

(c) $\left[\frac{4}{5}, 1\right]$ (d) $\left[\frac{5}{9}, 1\right]$

69. The coefficient of x^{203} in the expansion of $(x-1)(x^2-2)(x^3-3)\ldots(x^{20}-20)$ is

(a) -35 (b) 21

(c) 13 (d) 15

70. If the sum of n terms of the series $\frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots$ is S_n, then S_n exceeds 199 for all n greater than

(a) 99 (b) 50

(c) 199 (d) 100

71. The numbers $3^{2\sin 2x - 1}, 14, 3^{4 - 2\sin 2x}$ form first three terms of an AP, its fifth term is equal to

(a) -25 (b) -12

(c) 40 (d) 53

72. In a sequence of $(4n+1)$ terms the first $(2n+1)$ terms are in AP whose common difference is 2, and the last $(2n+1)$ terms are in GP whose common ratio is 0.5. If the middle terms of the AP and GP are equal, then the middle term of the sequence is

(a) $\frac{n \cdot 2^{2n+1}}{2.^{2n}-1}$ (b) $\frac{n \cdot 2^{n+1}}{2^{2n-1}}$

(c) $n2^n$ (d) none of these

73. If $a, a_1, a_2, a_3, \ldots, a_{2n}, b$ are in AP and $a, g_1, g_2, g_3, \ldots, g_{2n}, b$ are in GP and h is the HM of a and b, then

$$\frac{a_1+a_{2n}}{g_1 g_{2n}}+\frac{a_2+a_{2n-1}}{g_2 g_{2n-1}}+\ldots+\frac{a_n+a_{n+1}}{g_n g_{n+1}}$$

is equal to

(a) $\frac{2n}{h}$ (b) $2nh$

(c) nh (d) $\frac{n}{h}$

74. Let $S=\frac{8}{5}+\frac{16}{65}+\ldots+\frac{128}{2^{18}+1}$, then

(a) $S=\frac{1088}{545}$ (b) $S=\frac{545}{1088}$

(c) $S=\frac{1056}{545}$ (d) $S=\frac{545}{1056}$

75. If $\sin\theta, \sqrt{2}(\sin\theta+1), 6\sin\theta+6$ are in GP, then the fifth term is

(a) 81 (b) $82\sqrt{2}$

(c) 162 (d) none of these

76. If $\ln(a+c), \ln(c-a), \ln(a-2b+c)$ are in AP, then

(a) a, b, c are in AP (b) a^2, b^2, c^2 are in AP

(c) a, b, c are in GP (d) a, b, c are in HP

77. Let $a_1, a_2, a_3, \ldots$ be in an AP with common difference not a multiple of 3. Then maximum number of consecutive terms so that all are primes is

(a) 2 (b) 3

(c) 5 (d) infinite

78. The sum of the product taken two at a time of the numbers $1, 2, 2^2, 2^3, \ldots 2^{n-2}, 2^{n-1}$ is

(a) $\frac{1}{3}\cdot 2^{2n}+\frac{2}{3}$ (b) $\frac{1}{3}\cdot 2^{2n}-2^n+\frac{1}{3}$

(c) $\frac{1}{3}\cdot 2^{2n}-\frac{1}{3}$ (d) $\frac{1}{3}\cdot 2^{2n}-2^n+\frac{2}{3}$

79. The sum of the infinite terms of the series

$$\frac{5}{3^2\cdot 7^2}+\frac{9}{7^2\cdot 11^2}+\frac{13}{11^2\cdot 15^2}+\ldots \text{ is}$$

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$

(c) $\frac{1}{54}$ (d) $\frac{1}{72}$

80. Let a, b, c be positive real numbers, such that $bx^2+(\sqrt{(a+c)^2+4b^2})\,x+(a+c)\geq 0, \forall\, x \in R,$ then a, b, c are in

(a) GP (b) AP

(c) HP (d) none of these

81. $\left(1\frac{2}{3}\right)^2+\left(2\frac{1}{3}\right)^2+3^2+\left(3\frac{2}{3}\right)^2+\ldots$ to 10 terms, the sum is

(a) $\frac{1390}{9}$ (b) $\frac{1790}{9}$

(c) $\frac{1990}{9}$ (d) none of these

82. The consecutive odd integers whose sum is 45^2-21^2 are

(a) 43, 45,..., 75 (b) 43, 45,..., 79

(c) 43, 45,..., 85 (d) 43, 45,..., 89

83. If $<a_n>$ and $<b_n>$ be two sequences given by $a_n=(x)^{1/2^n}+(y)^{1/2^n}$ and $b_n=(x)^{1/2^n}-(y)^{1/2^n}$ for all $n \in N$, then $a_1\, a_2\, a_3 \ldots a_n$ is

(a) $\frac{x+y}{b_n}$ (b) $\frac{x-y}{b_n}$

(c) $\frac{x^2+y^2}{b_n}$ (d) $\frac{x^2-y^2}{b_n}$

84. If $x, |x+1|, |x-1|$ are the three terms of an AP. Its sum upto 20 terms is

(a) 90 or 175 (b) 180 or 350

(c) 360 or 700 (d) 720 or 1400

85. If $\Sigma n, \frac{\sqrt{10}}{3}\Sigma n^2, \Sigma n^3$ are in GP, then the value of n is

(a) 3 (b) 4

(c) 2 (d) non existent

86. If $\sum_{r=1}^{n} t_r=\frac{n(n+1)(n+2)(n+3)}{8}$, where t_r denotes the r th term of a series, then $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{t_r}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

87. If an AP, $a_7=9$ if $a_1\, a_2\, a_7$ is least, the common difference is

(a) $\frac{13}{20}$ (b) $\frac{23}{20}$

(c) $\frac{33}{20}$ (d) $\frac{43}{20}$

88. If the ratio of AM between two positive real numbers a and b to their HM is $m:n$; then $a:b$ is equal to

(a) $\frac{\sqrt{(m-n)}+\sqrt{n}}{\sqrt{(m-n)}-\sqrt{n}}$ (b) $\frac{\sqrt{n}+\sqrt{(m-n)}}{\sqrt{n}-\sqrt{(m-n)}}$

(c) $\frac{\sqrt{m}+\sqrt{(m-n)}}{\sqrt{m}-\sqrt{(m-n)}}$ (d) $\frac{\sqrt{(m-n)}+\sqrt{m}}{\sqrt{(m-n)}-\sqrt{m}}$

89. If $\sum_{n=1}^{k}\left[\frac{1}{3}+\frac{n}{90}\right]=21$, where $[x]$ denotes the integral part of x, then k is equal to

(a) 84 (b) 80
(c) 85 (d) none of these

90. If $x \in \{1, 2, 3, \ldots, 9\}$ and $f_n(x) = xxx \ldots x$ (n digits), then $f_n^2(3) + f_n(2)$ is equal to

(a) $2f_{2n}(1)$ (b) $f_n^2(1)$
(c) $f_{2n}(1)$ (d) $-f_{2n}(4)$

91. If three positive real numbers a, b, c are in AP with $abc = 4$, then minimum value of b is

(a) 4 (b) 3
(c) 2 (d) 1/2

92. The numbers of divisors of 1029, 1547 and 122 are in

(a) AP (b) GP
(c) HP (d) none of these

93. The coefficient of x^{15} in the product $(1-x)(1-2x)(1-2^2x)(1-2^3x)\ldots(1-2^{15}x)$ is

(a) $2^{105}-2^{121}$ (b) $2^{121}-2^{105}$
(c) $2^{120}-2^{104}$ (d) $2^{105}-2^{104}$

94. The roots of equation $x^2 + 2(a-3)x + 9 = 0$ lie between -6 and 1 and $2, h_1, h_2, \ldots, h_{20}, [a]$ are in HP, where $[a]$ denotes the integral part of a and $2, a_1, a_2, \ldots, a_{20}, [a]$ are in AP, then a_3h_{18} is equal to

(a) 6 (b) 12
(c) 3 (d) none of these

95. Value of $L = \lim_{n\to\infty}\frac{1}{n^4}\left[1\cdot\left(\sum_{k=1}^{n}k\right)+2\cdot\left(\sum_{k=1}^{n-1}k\right)+3\cdot\left(\sum_{k=1}^{n-2}k\right)+\ldots+n\cdot 1\right]$ is

(a) 1/24 (b) 1/12
(c) 1/6 (d) 1/3

Objective Questions Type II [One or more than one correct answer(s)]

In each of the questions below four choices of which one or more than one are correct. You have to select the correct answer(s) accordingly.

1. If $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in AP and x, y, z are also in AP (y being not equal to 0, 1 or -1), then

(a) x, y, z are in GP
(b) x, y, z are in HP
(c) $x = y = z$
(d) $(x-y)^2 + (y-z)^2 + (z-x)^2 = 0$

2. If d, e, f are in GP and the two quadratic equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then

(a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in HP (b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in GP
(c) $2dbf = aef + cde$ (d) $b^2df = ace^2$

3. If three unequal numbers p, q, r are in HP and their squares are in AP, then the ratio $p:q:r$ is

(a) $1-\sqrt{3}:-2:1+\sqrt{3}$ (b) $1:\sqrt{2}:-\sqrt{3}$
(c) $1:-\sqrt{2}:\sqrt{3}$ (d) $1+\sqrt{3}:-2:1-\sqrt{3}$

4. For a positive integer n, let $\alpha(n) = 1 + 1/2 + 1/3 + 1/4 + \ldots + 1/(2^n - 1)$, then

(a) $\alpha(100) \le 100$ (b) $\alpha(100) > 100$
(c) $\alpha(200) \le 100$ (d) $\alpha(200) > 100$

5. The pth term T_p of HP is $q(p+q)$ and qth term T_q is $p(p+q)$ when $p > 1, q > 1$, then

(a) $T_{p+q} = pq$ (b) $T_{pq} = p + q$
(c) $T_{p+q} > T_{pq}$ (d) $T_{pq} > T_{p+q}$

6. If the first and $(2n-1)$th term of an AP, a GP and a HP are equal and their nth terms are a, b and c respectively, then :

(a) $a = b = c$ (b) $a + c = b$
(c) $a \ge b \ge c$ (d) $ac = b^2$

7. If a, b, c be three unequal positive quantities in HP, then

(a) $a^{100} + c^{100} > 2b^{100}$ (b) $a^3 + c^3 > 2b^3$
(c) $a^5 + c^5 > 2b^5$ (d) $a^2 + c^2 > 2b^2$

8. The sum of n terms of the series $\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\ldots$ is

(a) $\frac{n(n^2+6n+11)}{18(n+1)(n+2)(n+3)}$

(b) $\frac{n^3+6}{18(n+1)(n+2)(n+3)}$

(c) $\frac{1}{18}-\frac{1}{3(n+1)(n+2)(n+3)}$

(d) $\frac{1}{6}-\frac{1}{2(n+1)(n+2)(n+3)}$

9. Given that $0 < x < \pi/4$ and $\pi/4 < y < \pi/2$ and $\sum_{k=0}^{\infty}(-1)^k\tan^{2k}x = a$, $\sum_{k=0}^{\infty}(-1)^k\cot^{2k}y = b$, then $\sum_{k=0}^{\infty}\tan^{2k}x\cot^{2k}y$ is

(a) $\frac{1}{a}+\frac{1}{b}-\frac{1}{ab}$ (b) $a + b - ab$

(c) $\frac{1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{ab}}$ (d) $\frac{ab}{a+b-1}$

10. If a, b, c are in HP, then the value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is

(a) $\frac{2}{bc}-\frac{1}{b^2}$ (b) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$

(c) $\frac{3}{b^2}-\frac{2}{ab}$ (d) none of these

11. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) Σn^2

(c) ${}^nC_3$ (d) ${}^{n+2}C_3$

12. If 1, $\log_y x$, $\log_z y$, $-15\log_x z$ are in AP, then

(a) $z^3 = x$ (b) $x = y^{-1}$

(c) $z^{-3} = y$ (d) $x = y^{-1} = z^3$

13. If b_1, b_2, b_3 ($b_1 > 0$) are three successive terms of a GP with common ratio r, the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by

(a) $r > 3$ (b) $r < 1$

(c) $r = 3.5$ (d) $r = 5.2$

14. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is equal to

(a) $\log_a(\log_b a)$

(b) $\log_a(\log_e a) - \log_a(\log_e b)$

(c) $-\log_a(\log_a b)$

(d) $\log_a(\log_e b) - \log_a(\log_e a)$

15. If a, b, c are in HP, then

(a) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in HP

(b) $\frac{2}{b} = \frac{1}{b-a} + \frac{1}{b-c}$

(c) $a-\frac{b}{2}, \frac{b}{2}, c-\frac{b}{2}$ are in GP

(d) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in HP

16. If $\sum_{r=1}^{n} r(r+1)(2r+3) = an^4 + bn^3 + cn^2 + dn + e$, then

(a) $a = 1/2$ (b) $b = 8/3$

(c) $c = 9/2$ (d) $e = 0$

17. If 1, $\log_9(3^{1-x}+2)$ and $\log_3(4\cdot 3^x - 1)$ are in AP, then x is equal to

(a) $\log_4 3$ (b) $\log_3 4$

(c) $1-\log_3 4$ (d) $\log_3(0.75)$

18. The series of natural numbers is divided into groups 1; 2, 3, 4; 5, 6, 7, 8, 9;.... and so on. Then the sum of the numbers in the nth group is

(a) $(2n-1)(n^2-n+1)$ (b) n^3-3n^2+3n-1

(c) $n^3+(n-1)^3$ (d) $n^3+(n+1)^3$

19. If $\alpha, \beta, \gamma, \delta$ are in AP and $\int_0^2 f(x)\,dx = -4$, where

$$f(x) = \begin{vmatrix} x+\alpha & x+\beta & x+\alpha-\gamma \\ x+\beta & x+\gamma & x-1 \\ x+\gamma & x+\delta & x-\beta+\delta \end{vmatrix}$$, then the common difference d is

(a) 1 (b) −1

(c) 2 (d) −2

20. If $a_i > 0$ for all $i \in N$, then

(a) $(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right) \geq 9$

(b) $\left(\frac{a_1}{a_2}+\frac{a_3}{a_4}+\frac{a_5}{a_6}\right)\left(\frac{a_2}{a_1}+\frac{a_4}{a_3}+\frac{a_6}{a_5}\right) \geq 9$

(c) $(a_1 a_2 + a_3 a_4)(a_1 a_3 + a_2 a_4) \geq 4a_1 a_2 a_3 a_4$

(d) none of the above

21. The pth, $2p$th and $4p$th terms of an AP are in GP the common ratio of GP is

(a) 2 (b) 1

(c) 4 (d) 1/2

22. If a, b, c are real and for some real x, $(a^2+b^2)x^2 + 2(ab+bc)x + (b^2+c^2) \leq 0$, then

(a) a, b, c are in GP

(b) a, b, c are in AP

(c) $ax^2 + 2bx + c \geq 0$ for all x

(d) $ax^2 + 2bx + c = 0$ for all x

23. Which of the following are true

(a) $x+\frac{1}{x} \geq 2$, if $x > 0$ (b) $x+\frac{1}{x} \leq -2$, if $x < 0$

(c) $2^x + 2^{-x} \geq 2$ for all x (d) $n! > n^n$

24. If a, b, c, d are distinct positive numbers in AP, then

(a) $ad < bc$

(b) $a + c < b + d$

(c) $a + d = b + c$

(d) $(a+1)(d+1) < (b+1)(c+1)$

25. The value of x for which $\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ are in AP lie in

(a) $(0, 1)$ (b) $(1, \infty)$

(c) $(0, \infty)$ (d) none of these

26. If a, b, c are in AP and a^2, b^2, c^2 are in HP, then

(a) $a = b = c$ (b) $a^2 = b^2 = c^2/2$

(c) a, b, c are in GP (d) $-a/2, b, c$ are in GP

27. If $\log_2(5\cdot 2^x + 1)$, $\log_4(2^{1-x}+1)$ and 1 are in AP, then x is equal to

(a) $\frac{\log 5}{\log 2}$ (b) $\log_2(0.4)$

(c) $1-\frac{\log 5}{\log 2}$ (d) $\frac{\log 2}{\log 5}$

28. If the arithmetic mean of two positive numbers a and b ($a > b$) is twice their geometric mean, then $a : b$ is

(a) $2+\sqrt{3} : 2-\sqrt{3}$ (b) $7+4\sqrt{3} : 1$

(c) $1 : 7-4\sqrt{3}$ (d) $2 : \sqrt{3}$

29. The determinant $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0$, if

(a) a, b, c are in AP (b) a, b, c are in GP

(c) a, b, c are in HP (d) $\alpha = \frac{1}{2}$

Linked-Comprehension Type

In these questions, a passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

Suppose p is the first of n $(n > 1)$ AM's between two positive numbers a and b; q the first of n HM's between the same two numbers.

On the basis of above information, answer the following questions :

1. The value of p is

(a) $\frac{na+b}{n+1}$ (b) $\frac{na-b}{n+1}$

(c) $\frac{nb+a}{n+1}$ (d) $\frac{nb-a}{n+1}$

2. The value of q is

(a) $\frac{ab\,(n+1)}{b+an}$ (b) $\frac{ab\,(n+1)}{(a+bn)}$

(c) $\frac{ab\,(n-1)}{b+an}$ (d) $\frac{ab\,(n-1)}{a+bn}$

3. The value of $\left(\frac{p}{q}-1\right)$ is

(a) $\frac{n}{(n-1)^2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2$ (b) $\frac{n}{(n-1)^2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2$

(c) $\frac{n}{(n+1)^2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2$ (d) $\frac{n}{(n+1)^2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2$

4. If $p = 8$ and $n = 3$, then

(a) q lies between 8 and 32

(b) q lies between 8 and 16

(c) q does not lies between 8 and 32

(d) q does not lies between 8 and 16

5. Final conclusion is

(a) q lies between p and $\left(\frac{n+1}{n-1}\right)p$

(b) q lies between p and $\left(\frac{n+1}{n-1}\right)^2 p$

(c) q does not lies between p and $\left(\frac{n+1}{n-1}\right)p$

(d) q does not lies between p and $\left(\frac{n+1}{n-1}\right)^2$.

PASSAGE 2

If A, G and H are respectively arithmetic, geometric and harmonic means between a and b both being unequal and positive, then $A = \frac{a+b}{2} \Rightarrow a+b = 2A, G = \sqrt{ab} \Rightarrow ab = G^2$ and $H = \frac{2ab}{a+b} \Rightarrow G^2 = AH$

From above discussion we can say that a, b are the roots of the equation $x^2 - 2A\,x + G^2 = 0$

Now, quadratic equation $x^2 - Px + Q = 0$ and quadratic equation $a\,(b-c)x^2 + b\,(c-a)x + c\,(a-b) = 0$ have a root common and satisfy the relation $b = \frac{2ac}{(a+c)}$, where a, b, c are real numbers.

On the basis of above information, answer the following questions :

1. The value of $\frac{a\,(b-c)}{b\,(c-a)}$ is

(a) −2 (b) 2

(c) −1 / 2 (d) 1/2

2. The value of [P] is (where [.] donotes the greatest integer function)

(a) −2 (b) −1

(c) 2 (d) 1

3. The value of [$2P - Q$] is (where [.] denotes the greatest integer function)

(a) 2 (b) 3

(c) 5 (d) 6

4. If the geometric and harmonic means of two numbers are 16 and $12\frac{4}{5}$, then the ratio of one number to the other is

(a) 1 : 4 (b) 2 : 3

(c) 1 : 2 (d) 2 : 1

5. The sum of the AM and GM of two positive numbers is equal to the difference between the numbers. The numbers are in the ratio

(a) 1 : 3 (b) 1 : 6

(c) 9 : 1 (d) 1 : 12

6. The ratio of the AM, GM and HM of the roots of the given quadratic equation is

(a) 1 : 2 : 3 (b) 1 : 1 : 2

(c) 2 : 2 : 3 (d) 1 : 1 : 1

PASSAGE 3

If a sequence or series is not a direct form of an AP, GP, etc. Then its nth term can not be determined. In such cases, we use the following steps to find the nth term (T_n) of the given sequence.

Step -I : Find the differences between the successive terms of the given sequence. If these differences are in AP, then take $T_n = an^2 + bn + c$, where a, b, c are constants.

Step-II : If the successive differences finding in step I are in GP with common ratio r, then take

$T_n = a + bn + cr^{n-1}$, where a, b, c are constants.

Step -III : If the second successive differences (Differences of the differences) in step I are in AP, then take

$T_n = an^3 + bn^2 + cn + d$, where a, b, c, d are constants.

Step-IV : If the second successive differences (Differences of the differences) in step I are in GP, then take

$T_n = an^2 + bn + c + dr^{n-1}$, where a, b, c, d are constants.

Now let sequences :

A : 1, 6, 18, 40, 75, 126, ...   B : 1, 1, 6, 26, 91, 291, ...   C : ln 2 ln 4, ln 32, ln 1024, ...

On the basis of above information, answer the following questions :

1. Second successive differences of the sequences A and B are :
(a) Both AP   (b) Both GP
(c) AP and GP respectively   (d) GP and AP respectively

2. If the nth term (T_n) of the sequence A is $an^3 + bn^2 + cn + d$, then $6a + 2b - d$ is
(a) ln 2   (b) 2
(c) ln 8   (d) 4

3. The format of nth term (T_n) of the sequence C is
(a) $an^2 + bn + c$ (b) $an^3 + bn^2 + cn + d$
(c) $an + b + cr^{n-1}$ (d) $an^2 + bn + c + dr^{n-1}$

4. The correct statement for sequence B is
(a) second successive differences makes AP with common difference 3
(b) second successive differences makes AP with common differences ln 4
(c) second successive differences makes a GP with common ratio 3
(d) second successive differences makes a GP with common ratio 4

5. The correct statement for sequence C is
(a) First successive differences form an AP with common difference ln 4
(b) First successive differences form a GP with common ratio 4
(c) Second successive differences form an AP with common difference ln 2
(d) Second successive differences form a GP with common ratio 2

PASSAGE 4

The sum of the squares of three distinct real numbers which are in strictly increasing GP is S^2. If their sum is αS.

On the basis of above information, answer the following questions :

1. α^2 lies in
(a) $\left(\frac{1}{3}, 1\right)$ (b) (1, 2)
(c) $\left(\frac{1}{3}, 3\right)$ (d) $\left(\frac{1}{3}, 1\right) \cup (1, 3)$

2. If $\alpha^2 = 2$, then the value of $[r]$ is (where [.] denotes the greatest integer function and r is common ratio of GP)
(a) 0   (b) 1
(c) 2   (d) 3

3. If $r = 2$, then the value of (α^2) is (where (.) denotes the least integer function and r is common ratio of GP)
(a) 0   (b) 1
(c) 2   (d) 3

4. If $S = 10\sqrt{3}$, then the greatest value of the middle term is
(a) 5   (b) $5\sqrt{3}$
(c) 10   (d) $10\sqrt{3}$

5. If we drop the condition that the GP is strictly increasing and take $r^2 = 1$, (where r is common ratio of GP) then the value of α is
(a) 0   (b) ± 1
(c) ± 2 (d) $\pm\sqrt{3}$

PASSAGE 5

We are giving the concept of arithmetic mean of mth power. Let $a, b > 0$ and $a \neq b$ and let m be a real number. Then

$$\frac{a^m + b^m}{2} > \left(\frac{a+b}{2}\right)^m, \text{ if } m \in R \sim [0, 1]$$

However if $m \in (0, 1)$, then $\frac{a^m + b^m}{2} < \left(\frac{a+b}{2}\right)^m$

Obviously if $m \in \{0, 1\}$, then $\frac{a^m + b^m}{2} = \left(\frac{a+b}{2}\right)^m$

On the basis of above information, answer the following questions :

1. If a, b be positive and $a + b = 1$ $(a \neq b)$ and if $A = \sqrt[3]{a} + \sqrt[3]{b}$ then the correct statement is

(a) $A > 2^{2/3}$ (b) $A = \frac{2^{2/3}}{3}$

(c) $A < 2^{2/3}$ (d) $A = 2^{2/3}$

2. If x, y be positive real numbers such that $x^2 + y^2 = 8$, then the maximum value of $x + y$ is

(a) 2 (b) 4

(c) 6 (d) 8

3. If a, b, c are positive real numbers but not all equal such that $a + b + c = 1$, then best option of values $\frac{b^2 + c^2}{b+c} + \frac{c^2 + a^2}{c+a} + \frac{a^2 + b^2}{a+b}$ lie between

(a) $\left(\frac{3}{2}, \infty\right)$ (b) $(1, \infty)$

(c) $(0, \infty)$ (d) none of these

4. If a and b are positive $(a \neq b)$ and $a + b = 1$ and if $A = \left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2$, then the correct statement is

(a) $A > 8$ (b) $A < 8$

(c) $A > \frac{25}{2}$ (d) $A < \frac{25}{2}$

5. If a, b, c be positive and in Harmonic progression and if $\lambda = \frac{a^n + c^n}{b^n}, \forall n \notin (0, 1)$, then the correct statement is

(a) $\lambda < 2$ (b) $\lambda > 2$

(c) $\lambda = 2$ (d) none of these

PASSAGE 6

We are giving the concept of arithmetic mean of m th power. Let $a_1, a_2, a_3, \ldots, a_n$ be n positive real numbers (not all equal) and let m be real number.

Then $\frac{a_1^m + a_2^m + a_3^m + \ldots + a_n^m}{n} > \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^m$; if $m \in R \sim [0,1]$

However if $m \in (0,1)$, then $\frac{a_1^m + a_2^m + a_3^m + \ldots + a_n^m}{n} < \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^m$

Obviously if $m \in \{0, 1\}$, then $\frac{a_1^m + a_2^m + a_3^m + \ldots + a_n^m}{n} = \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^m$

On the basis of above information, answer the following questions :

1. If sum of the mth powers of first n odd numbers is λ, $\forall\, m > 1$ then

(a) $\lambda < n^m$ (b) $\lambda > n^m$

(c) $\lambda < n^{m+1}$ (d) $\lambda > n^{m+1}$

2. If a, b, c be positive real numbers, then the possible best option of values $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ lie between

(a) $(1, \infty)$ (b) $(0, \infty)$

(c) $\left[\frac{3}{2}, \infty\right)$ (d) $[2, \infty)$

3. If $x > 0, y > 0, z > 0$ and $x + y + z = 1$, then the minimum value of $\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z}$ is

(a) $\frac{9}{5}$ (b) $\frac{8}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{5}$

4. If $a_1, a_2, a_3, \ldots, a_n$ are all positive such that $a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2 = A$ and if greatest and least values of $(a_1 + a_2 + a_3 + \ldots + a_n)^2$ are G and L respectively, then

(a) $G - L = 2A$
(b) $G - L = nA$
(c) $G - L = (n-1)A$
(d) $G - L = (n-2)A$

5. If a, b, c, d be positive and not all equal to one another such that $a + b + c + d = 4/3$, then the minimum value of

$$\sqrt{\left(\frac{1}{b+c+d}\right)} + \sqrt{\left(\frac{1}{c+d+a}\right)} + \sqrt{\left(\frac{1}{d+a+b}\right)} + \sqrt{\left(\frac{1}{a+b+c}\right)}$$ is

(a) 2
(b) 4
(c) 6
(d) 8

PASSAGE 7

If $a_i > 0, i = 1, 2, 3, \ldots, n$ and $m_1, m_2, m_3, \ldots, m_n$ be positive rational numbers, then

$$\left(\frac{m_1a_1 + m_2a_2 + \ldots + m_na_n}{m_1 + m_2 + \ldots + m_n}\right) \geq (a_1^{m_1} a_2^{m_2} \ldots a_n^{m_n})^{1/(m_1 + m_2 + \ldots + m_n)}$$

$$\geq \frac{(m_1 + m_2 + \ldots + m_n)}{\frac{m_1}{a_1} + \frac{m_2}{a_2} + \ldots + \frac{m_n}{a_n}}$$ is called weighted mean theorem

where $A^* = \dfrac{m_1a_1 + m_2a_2 + \ldots + m_na_n}{m_1 + m_2 + \ldots + m_n}$

= Weighted arithmetic mean

$G^* = (a_1^{m_1} a_2^{m_2} \ldots a_n^{m_n})^{1/(m_1 + m_2 + \ldots + m_n)}$

= Weighted geometric mean

and $H^* = \dfrac{m_1 + m_2 + \ldots + m_n}{\frac{m_1}{a_1} + \frac{m_2}{a_2} + \ldots \frac{m^n}{a_n}}$ = Weighted harmonic mean

ie., $A^* \geq G^* \geq H^*$

Now, let $a + b + c = 5\ (a, b, c > 0)$ and $x^2y^3 = 6\ (x > 0, y > 0)$

On the basis of above information, answer the following questions :

1. The greatest value of ab^3c is

(a) 3
(b) 9
(c) 27
(d) 81

2. Which statement is correct

(a) $\dfrac{1}{5} \geq \dfrac{1}{\frac{1}{a} + \frac{3}{b} + \frac{1}{c}}$

(b) $\dfrac{1}{25} \geq \dfrac{1}{\frac{1}{a} + \frac{9}{b} + \frac{1}{c}}$

(c) $\dfrac{1}{5} \geq \dfrac{1}{\frac{1}{a} + \frac{9}{b} + \frac{1}{c}}$

(d) $\dfrac{1}{25} \geq \dfrac{1}{\frac{1}{a} + \frac{6}{b} + \frac{1}{c}}$

3. The least value of $3x + 4y$ is

(a) 5
(b) 7
(c) 10
(d) 17

4. Which statement is correct

(a) $\dfrac{2x+3y}{5} \geq (6)^{1/5} \geq \dfrac{5}{\frac{3}{x} + \frac{2}{y}}$

(b) $\dfrac{2x+3y}{5} \geq (6)^{1/5} \geq \dfrac{5xy}{3x+2y}$

(c) $\dfrac{2x+3y}{5} \geq (6)^{1/5} \geq \dfrac{5xy}{3x+4y}$

(d) $\dfrac{2x+3y}{5} \geq (6)^{1/5} \geq \dfrac{5xy}{2x+3y}$

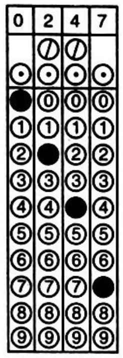
5. The maximum value of $\lambda \mu \upsilon$ when $\dfrac{\lambda^2}{d_1^2} + \dfrac{\mu^2}{d_2^2} + \dfrac{\upsilon^2}{d_3^2} = 1$ is

(a) $\dfrac{d_1d_2d_3}{\sqrt[3]{3}}$
(b) $\dfrac{d_1d_2d_3}{3\sqrt{3}}$
(c) $\dfrac{d_1d_2d_3}{3}$
(d) $\dfrac{d_1d_2d_3}{\sqrt{3}}$

Numerical Grid-Based Problems

Solve the following problems and mark your response against their respective grids. Write your answer in the top row of the grid and darken the concerned numbers in the respective columns.

For example. If answer of a question is 0247, then



1. If $x_1, x_2, x_3, \ldots, x_{2008}$ are in HP and $\sum_{i=1}^{2007} x_i\, x_{i+1} = \lambda\, x_1 x_{2008}$, then λ must be equal to :

2. If $S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{n-1}}$ and $2 - S_n < \frac{1}{100}$, then the least value of n must be :

3. If (1)(2010) + (2) (2009) + (3) (2008) + ... + (2010) (1) = (335) (2011)(λ), then the value of λ must be

4. A three digit number whose consecutive digits form a GP. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, the resulting digits will form an AP. The three digit number must be

5. If $\sum_{r=1}^{n} T_r = \frac{n\,(n+1)(n+2)(n+3)}{8}$, then the value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{4016}{T_r}$ must be

6. If $S_n = 1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + \ldots\ldots \ldots = \frac{n\,(n+1)^2}{2}$, where n is even, then the value of S_{21} must be

7. If a_n denotes the coefficient of x^n in $P(x) = (1 + x + 2x^2 + 3x^3 + \ldots + n\,x^n)^2$, then the value of a_{24} must be

8. The sum of the products taken two at a time of the numbers 1, 2, 3, 4, ..., 10 is

9. If a, b, c are in HP and if $\left(\frac{a+b}{2a-b}\right) + \left(\frac{c+b}{2c-b}\right) > \sqrt{\lambda\sqrt{\lambda\sqrt{\lambda \ldots \infty}}}$, then the value of λ must be

10. The sum of two digit even numbers which do not end with zero is

11. The number of zeros in the end in the product of $5^6 \times 6^7 \times 7^8 \times 8^9 \times \ldots \ldots \times 50^{51}$ must be

12. If $\log_{10} x + \frac{1}{2}\log_{10} x + \frac{1}{4}\log_{10} x + \ldots = y$ and $\frac{1+3+5+\ldots+(2y-1)}{4+7+10+\ldots+(3y+1)} = \frac{20}{7\log_{10} x}$ $\forall\, x, y \in N$, then the value of $(\log_y x)^5$ must be

13. Two consecutive numbers from $1, 2, 3, \ldots, n$ are removed, then arithmetic mean of the remaining numbers is $\frac{105}{4}$, then n must be equal to

14. Let a, b be positive real numbers. If a, A_1, A_2, b are in AP. a, G_1, G_2, b are in GP and a, H_1, H_2, b are in HP, then $\frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(\lambda a + b)(a + \lambda b)}{\mu\, ab}$, then the value of $\lambda^\mu + \mu^\lambda$ must be

15. If $S = \sum_{r=1}^{90} \frac{r}{(r^4 + r^2 + 1)}$, then the value of 8191S must be

16. Balls are arranged in rows to form an equilateral triangle. The first row consist of one ball, the second row of two balls, the third row of three balls and so on. If 669 more balls are added then all the balls can be arranged in the shape of a square and each of the sides, then contains 8 balls less than each side of the triangle, then the initial no. of balls must be

Matrix-Match Type

Given below are Matching Type Questions, with two columns (each having some items) each . Each item of Column I has to be matched with the items of Column II, by encircling the correct match(es).

NOTE An item of Column I can be matched with more than one items of Column II. All the items of Column II have to be matched.

1. **Observe the following columns :**

	Column I		Column II
(A)	If a, b, c are non-zero real numbers such that $3(a^2+b^2+c^2+1) = 2\times(a+b+c+ab+bc+ca)$, then a, b, c are in	(P)	AP
(B)	If the square of differences of three numbers be in AP, then their differences are in	(Q)	GP
(C)	If $a-b, ax-by, ax^2-by^2 (a, b \neq 0)$ are in GP, then $x, y, \frac{ax-by}{a-b}$ are in	(R)	HP
		(S)	Equal

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

Answers

Objective Questions Type I [Only one correct answer]

1. (b)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (c)
21. (b)	**22.** (a)	**23.** (d)	**24.** (b)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (d)
31. (d)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (d)	**38.** (c)	**39.** (d)	**40.** (a)
41. (d)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (a)	**48.** (c)	**49.** (a)	**50.** (c)
51. (c)	**52.** (c)	**53.** (d)	**54.** (c)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (a)	**59.** (b)	**60.** (a)
61. (b)	**62.** (a)	**63.** (b)	**64.** (a)	**65.** (c)	**66.** (a)	**67.** (c)	**68.** (b)	**69.** (c)	**70.** (c)
71. (d)	**72.** (a)	**73.** (a)	**74.** (a)	**75.** (c)	**76.** (d)	**77.** (d)	**78.** (d)	**79.** (d)	**80.** (b)
81. (d)	**82.** (d)	**83.** (b)	**84.** (b)	**85.** (b)	**86.** (c)	**87.** (c)	**88.** (c)	**89.** (b)	**90.** (c)
91. (a)	**92.** (a)	**93.** (a)	**94.** (b)	**95.** (a)					

Objective Questions Type II [One or more than one correct answer(s)]

1. (a, b, c, d)	**2.** (a, c)	**3.** (a, d)	**4.** (a, d)	**5.** (a, b, c)
6. (c, d)	**7.** (a, b, c, d)	**8.** (a, c)	**9.** (c, d)	**10.** (a, b, c)
11. (a, d)	**12.** (a, b, c, d)	**13.** (a, b, c, d)	**14.** (a, b)	**15.** (a, b, c, d)
16. (a, b, c, d)	**17.** (c, d)	**18.** (a,c)	**19.** (a, b)	**20.** (a, b, c)
21. (a, b)	**22.** (a,c)	**23.** (a, b, c)	**24.** (a, c, d)	**25.** (a, b)
26. (a, c, d)	**27.** (b, c)	**28.** (a, b, c)	**29.** (b, d)	

Linked-Comprehension Type

Passage 1 **1.** (a) **2.** (b) **3.** (c) **4.** (c) **5.** (d)

Passage 2 **1.** (c) **2.** (c) **3.** (b) **4.** (a) **5.** (c) **6.** (d)

Passage 3 **1.** (c) **2.** (d) **3.** (a) **4.** (c) **5.** (a)

Passage 4 **1.** (d) **2.** (c) **3.** (d) **4.** (c) **5.** (d)

Passage 5 **1.** (c) **2.** (b) **3.** (b) **4.** (c) **5.** (b)

Passage 6 **1.** (d) **2.** (c) **3.** (c) **4.** (c) **5.** (b)

Passage 7 **1.** (c) **2.** (c) **3.** (c) **4.** (b) **5.** (b)

Numerical Grid-Based Problems

1. 2 0 0 7	**2.** 0 0 0 8	**3.** 2 0 1 2	**4.** 0 9 3 1
5. 2 0 0 8	**6.** 4 8 5 1	**7.** 2 6 0 0	**8.** 1 3 2 0
9. 0 0 0 4	**10.** 1 9 8 0	**11.** 0 2 8 5	**12.** 3 1 2 5
13. 0 0 5 0	**14.** 0 5 9 3	**15.** 4 0 9 5	**16.** 1 5 4 0

Matrix-Match Type

1. A → (P, Q, S); B → (R); C → (P, Q, S)